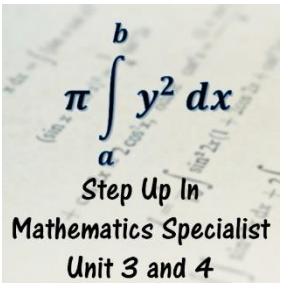
3.3 The Cross Product and Planes

Problems Worksheet



1. Determine a vector perpendicular to both vectors in the following pairs.

   a. $\boldsymbol{u} = 2\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{v} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$

   b. $\boldsymbol{e} = 3\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{f} = \boldsymbol{i}$

   c. $\boldsymbol{j}$ and $\boldsymbol{k}$

   d. $\boldsymbol{j}$ and $2\boldsymbol{k}$

2. Explain the following.

   a. In two dimensions, $\boldsymbol{r}.\boldsymbol{n} = c$ is one form of the vector equation of a line, whilst in three dimensions $\boldsymbol{r}.\boldsymbol{n} = c$ is the vector equation of a plane. Use at least one diagram to aid your explanation.

   b. In three dimensions, the following two vector equations of a plane are identical:
   $\boldsymbol{r}.(3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) = 5$ and $\boldsymbol{r}.(-6\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}) = -10$

3. Consider the unique plane which passes through the points with position vectors $A\ (3\boldsymbol{i} - 2\boldsymbol{k})$, $B\ (\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$ and $C\ (-4\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$. Determine the equation of this plane in the following forms:

   a. $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$

   b. $\boldsymbol{r}.\boldsymbol{n} = c$

   c. $ax + by + cz = d$

   d. Confirm your answer to part (c) with use of a ClassPad eActivity.

4. Determine the shortest distance between the following planes and points. Ensure you are able to use a ClassPad eActivity to determine the solutions to all questions below.

   a. The origin and the plane $\boldsymbol{r} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$.

   b. The origin and the plane $-x + 2y - 4z = 16$.

   c. The point with position vector $3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and the plane $\boldsymbol{r} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$.

   d. The point with position vector $3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and the plane $\boldsymbol{r}.(-2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}) = -7$.

5. Determine the separation distance between the following parallel planes. Ensure you are able to use a ClassPad eActivity to assist determining the solutions to all questions below.

a. $\boldsymbol{r}.\begin{pmatrix}1\\1\\2\end{pmatrix}=5$ and $\boldsymbol{r}.\begin{pmatrix}1\\1\\2\end{pmatrix}=8.$

b. $\boldsymbol{r}.\begin{pmatrix}-3\\2\\3\end{pmatrix}=5$ and $\boldsymbol{r}.\begin{pmatrix}6\\-4\\-6\end{pmatrix}=12.$

c. $\boldsymbol{r}=\begin{pmatrix}3+2\lambda-\mu\\-1-\lambda+\mu\\4+\lambda-3\mu\end{pmatrix}$ and $\boldsymbol{r}=\begin{pmatrix}-8+2\lambda-\mu\\1-\lambda+\mu\\10+\lambda-3\mu\end{pmatrix}.$

d. $\boldsymbol{r}=\begin{pmatrix}4\\1\\6\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\2\end{pmatrix}$ and $x+5y+z=19.$

6. Determine the parametric, vector and Cartesian equations of the line that represents the intersection of the following pairs of planes.

   a. $\boldsymbol{r}.(2\boldsymbol{i}-\boldsymbol{j}+8\boldsymbol{k})=14$ and $\boldsymbol{r}.(\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k})=7$.

   b. $\boldsymbol{r}=\begin{pmatrix}1\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\0\\-3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\2\end{pmatrix}$ and $\boldsymbol{r}.\begin{pmatrix}-1\\1\\3\end{pmatrix}=3$.